Baroclinic eddies and the extent of the Hadley circulation: An idealized GCM study

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The Hadley Circulation has widened over the past 30 years. This widening has been qualitatively reproduced in general circulation model (GCM) simulations of a warming climate. Comprehensive GCM studies suggest this widening may be caused by a poleward shift in baroclinic eddy activity. Yet the limited amplitude of the climate change signals analyzed precludes a quantitative comparison with theories.

This study uses two idealized GCMs, one with and one without an active hydrologic cycle, to investigate changes in the extent of the Hadley circulation over a range of climates, which span global-mean temperatures from 243 to 385 K and pole-to-equator temperature contrasts from 12 to 100 K. Baroclinic eddies control the extent of the Hadley circulation across most of this wide range of climates. A supercriticality criterion that quantifies the depth of baroclinic eddies relative to that of the troposphere turns out to be a good indicator of where baroclinic eddies become deep enough to terminate the Hadley circulation. The supercriticality depends on meridional temperature gradients and an effective stability, which accounts for the effect of convective heating on baroclinic eddies.

As the pole-to-equator temperature contrast weakens or the local convective static stability increases, convective heating increasingly influences the thermal stratification of the troposphere and the supercriticality. Consistent with the supercriticality criterion, the Hadley circulation contracts as meridional temperature gradients increase, and it widens as the effective static stability increases. The former occurs during El Niño and may account for the observed Hadley circulation contraction then; the latter occurs during global warming.
1. Introduction

Observational studies suggest that the Hadley circulation in the tropics has widened over the past 30 years (Hu and Fu 2007; Seidel and Randel 2007; Seidel et al. 2008; Adam et al. 2014). This widening has been qualitatively reproduced in general circulation model (GCM) simulations of global warming scenarios; it is accompanied by a poleward shift of the subtropical dry zones (e.g., Lu et al. 2007) and midlatitudes storm tracks (e.g., Yin 2005). At the same time, the Hadley circulation has been observed to contract during El Niño and expand during La Niña (e.g., Nguyen et al. 2013).

There is no convincing account of what causes the widening trend or ENSO variations of the Hadley circulation; a theory describing the width of the Hadley circulation on an Earth-like planet is lacking (Schneider 2006; Schneider et al. 2010). The now-prevalent hypothesis is that the Hadley circulation terminates where baroclinic eddies in some sense start controlling the dynamics. Simple versions of this argument are based on the linear stability criterion for the quasi-geostrophic (QG) two-layer model and state that the Hadley circulation extends up to the latitude at which angular momentum–conserving axisymmetric flow becomes baroclinically unstable (Held 2000). Such arguments have been widely used in the recent literature to account for the poleward shift of the Hadley circulation terminus in comprehensive GCM simulations (Lu et al. 2007; Frierson et al. 2007). Yet the resulting Hadley circulation extent compares poorly with idealized GCM simulations over a much wider range of climates than that sampled in the comprehensive GCM simulations (Walker and Schneider 2006; Korty and Schneider 2008). Moreover, its core assumptions—angular momentum–conserving flow in the upper branch of the Hadley circulation, and the existence of a critical shear for baroclinic instability—are violated in Earth’s atmosphere (Schneider 2006; Zurita-Gotor and Lindzen 2007). A modified version of this argument is based
on a diffusive eddy mixing model and posits that the Hadley circulation extends to the latitude where baroclinic eddies become deep enough to reach the upper troposphere and where, as a consequence, the eddy flux of angular momentum changes sign (Schneider and Walker 2006; Korty and Schneider 2008). Determining this latitude from a supercriticality criterion accounts broadly for Hadley circulation changes in simulations with a dry idealized GCM (Korty and Schneider 2008). But applying this criterion to an atmosphere with an active hydrologic cycle has remained challenging (Schneider and O’Gorman 2008). More recently, O’Gorman (2011) suggested a way to include moisture effects in the depth scaling of baroclinic eddies. This modified scaling was found to capture well changes in the extent of the Hadley circulation in global warming simulations with an idealized GCM with an active hydrologic cycle (O’Gorman 2011). However, some important discrepancies with the dry GCM simulations of Walker and Schneider (2006) and Korty and Schneider (2008) were left unexplained.

Here we describe a modified supercriticality criterion, which is very similar to that defined by O’Gorman (2011), and we test its relevance over a wide range of climates simulated with two idealized GCMs. We show that this criterion can discriminate between regions dominated by baroclinic eddy or convective activity, and that it constrains changes in the Hadley circulation extent with climate when baroclinic wave activity is strong in the extratropics.

2. Heuristic arguments

The simplest model of the Hadley circulation is that of an axisymmetric overturning circulation extending to a finite latitude, beyond which radiative–convective equilibrium prevails (Schneider 1977; Held and Hou 1980). Assuming an angular momentum–conserving circulation that is energetically closed, Held and Hou (1980) showed that such a Hadley circulation, in the approximation...
of small latitudes, would extend up to the latitude

\[ \phi_{HH} = \left( \frac{\Delta h \theta_R}{\theta_0} \frac{gH_t}{\Omega^2 a^2} \right)^{1/2}. \]  

(1)

Here, \( g \) is the gravitational acceleration, \( \Omega \) the angular velocity of the planetary rotation, \( a \) the planetary radius, and \( \theta_0 \) is a constant reference value of potential temperature. Radiative–equilibrium temperatures near the equator are assumed to decrease quadratically with latitude, and \( \Delta h \theta_R \) is the pole-to-equator radiative–equilibrium temperature difference; \( H_t \) is the height of the upper branch of the Hadley circulation (i.e., approximately the tropical tropopause height).

Such axisymmetric flows were not originally viewed as accounting for the behavior of Earth’s Hadley circulation, but as providing a basic state for studies of baroclinic instability (e.g., Schneider 1977). The zonal wind speeds consistent with an angular momentum–conserving mean flow would be large enough in the subtropics to be linearly unstable to baroclinic instability (e.g., Phillips 1954). This led to the notion that the axisymmetric Hadley circulation may extend up to the latitude where baroclinic instability “sets in” (Held 2000). More generally, the Hadley circulation terminus may be considered as the equatorward boundary of the region where baroclinic eddies become deep enough to reach the upper troposphere (see Fig. 1). Where they reach the upper troposphere, wave activity no longer propagates upward but horizontally, implying angular momentum flux convergence in the latitude band where it is generated (e.g., Vallis 2006). Because the Rossby number in the descending branch of the Hadley circulation and poleward is generally small (e.g., Walker and Schneider 2006; Levine and Schneider 2011), this implies equatorward mean meridional flow where baroclinic eddies are generated and reach the upper troposphere. In this view, the terminus of the Hadley circulation defines the equatorward margin of wave activity generation and upward propagation into the upper troposphere (Korty and Schneider 2008).
Building on Held (1978), Schneider and Walker (2006) derived a non-dimensional criterion, called supercriticality

\[ S_c = \frac{\Delta h \theta}{\Delta v \theta} \approx \frac{\bar{p}_o - \bar{p}_e}{\bar{p}_o - \bar{p}_t}, \]  

which quantifies the ratio of the depth scale of baroclinic entropy fluxes to the height of the tropopause in dry atmospheres. Here, \( \langle \cdot \rangle \) defines a temporal and zonal mean; \( \bar{p}_o \) is the mean surface pressure, \( \bar{p}_t \) is the mean tropopause pressure (determined as described in the appendix), and \( \bar{p}_e \) is the mean pressure to which substantial eddy entropy fluxes extend; \( \Delta h \theta \) is a meridional potential temperature contrast, defined with the near-surface meridional gradient \( \partial_y \bar{\theta}_s \) of potential temperature as

\[ \Delta h \theta = -\frac{f}{\beta} \partial_y \bar{\theta}_s, \]  

and \( \Delta v \theta \) is a near-surface static stability measure, defined with the near-surface static stability \(-\partial_p \bar{\theta}_s\) as

\[ \Delta v \theta = -2 \partial_p \bar{\theta}_s (\bar{p}_o - \bar{p}_t). \]  

The near-surface average \( \langle \cdot \rangle_s \) is taken to be the horizontal mean averaged between the 800- and 700-hPa pressure levels, that is, just above the planetary boundary layer.

In regions where baroclinic wave activity strongly affects the mean flow and thus sets the depth of the troposphere, we have \( S_c \approx 1 \). In regions where baroclinic eddies are shallower than the depth of the troposphere, \( S_c < 1 \). The latter occurs where radiative-convective processes set the tropopause height. To obtain relation (2) for the depth of baroclinic eddy entropy fluxes, Schneider and Walker (2006) assumed that potential vorticity along dry isentropes is mixed in the troposphere, that potential temperature is mixed along the surface, and that the turbulent diffusivity has no essential vertical structure, i.e., the mixing is barotropic over the depth over which baroclinic
eddies extend. The latter is expected to be the case in the limit of strong turbulence and is an
approximation when turbulence is weak (cf. Held 1978). Relation (2) is strictly valid only when
applied on length scales greater than that of the eddies, which are often of planetary scale (see the
appendix for a definition of this mixing region).

The supercriticality defined in relation (2) is formally similar to an expression derived in the
QG two-layer model comparing the wind shear to its critical value for neutrality with respect to
linear baroclinic instability. This QG scaling was applied to Earth observations by Stone (1978)
and Stone and Nemet (1996) to suggest that Earth’s extratropical troposphere is close to neutral
with respect to linear baroclinic instability. Others have used this linear stability interpretation
to explain poleward shifts in the terminus of the Hadley circulation with global warming (e.g.,
Held 2000; Vecchi and Soden 2007; Frierson et al. 2007). Despite the formal similarity, the in-
terpretations of the QG expression and the supercriticality (2) differ. The supercriticality (2) does
not assume or imply a state of marginal stability, which is known not to be attained in the atmo-
sphere (e.g., Zurita-Gotor and Lindzen 2007; Merlis and Schneider 2009). Rather, the result $S_c \lesssim 1$
emerges from considering the limit of strong turbulence and how it would affect the atmospheric
thermal structure.

Using an idealized dry GCM like that we use here, Schneider and Walker (2006) found that $S_c$,
when averaged over regions of strong baroclinic eddy activity, assumed $O(1)$ values over a wide
range of climates. Values $S_c \sim 1$ were attained when the tropospheric thermal stratification was
controlled by baroclinic eddies. Values $S_c < 1$ were attained when convection stabilized the tro-
pospheric thermal stratification more than baroclinic eddies could, that is, when baroclinic eddies
were relatively weak. (Values $S_c \gtrsim 1$ were also obtained, when baroclinic eddies were weak and
convection lead to a nearly dry-neutral thermal stratification, rendering the supercriticality poorly
defined. This is the regime recently explored in more detail by Jansen and Ferrari (2013).) These
results suggest one may use $S_c$ to locate the terminus of the Hadley circulation in climates in which baroclinic eddies influence the thermal stratification (see Fig. 1). Using $S_c$ as a local-in-latitude measure of the depth of baroclinic entropy fluxes, Korty and Schneider (2008) found that the Hadley circulation in many climates extends to the latitude at which $S_c$ first exceeds a critical $O(1)$ value. ($S_c$ generally increases with latitude because the geometric term $f/\beta = a\tan \phi$ increases rapidly with latitude $\phi$.) Here we build on that work and extend the results to regimes in which convection and latent heat release are important in controlling the subtropical and extratropical thermal stratification.

**b. Effective static stability**

Schneider and Walker (2006) derived the supercriticality (2) neglecting moisture effects on baroclinic eddies. However, latent heat release in large-scale condensation and moist convection are known to affect the intensity and structure of baroclinic eddies (e.g., Emanuel et al. 1987; Gutowski Jr. et al. 1992; Lapeyre and Held 2004), as well as the tropospheric thermal stratification (Korty and Schneider 2007). Using an idealized moist GCM, Schneider and O’Gorman (2008) found a considerable effect of moisture on $S_c$: as the climate warmed and moisture effects became more prevalent, the supercritality decreased to values $S_c < 1$, in contrast to its near-invariance found in the dry GCM simulations. Only in the coldest moist simulations were $O(1)$ values of supercriticality observed. Schneider and O’Gorman (2008) attributed this decrease with global warming to an increase in the static stability and tropopause height, associated with increasing surface temperature and large-scale condensation in moist baroclinic eddies. This points to the need for a modified static stability measure that accounts for latent heat release in baroclinic eddies. Such effective static stability was proposed by O’Gorman (2011), whose ideas we briefly review here.
To devise a theory for an effective static stability that takes latent heat release in eddies into account, O'Gorman (2011) addresses the asymmetry that exists in moist atmospheres between updrafts, which favor moisture saturation and latent heat release, and downdrafts without condensation. This asymmetry affects the atmospheric dry entropy budget

$$\frac{\partial t}{\partial \theta} + \ldots + \omega \frac{\partial \rho \theta}{\partial p} = \ldots + Q_c.$$  \hspace{2cm} (5)

Here, $Q_c$ refers to latent heat release associated with condensation of water, which dominates locally over other diabatic processes such as radiative heating when condensation occurs. Changes in the dry entropy of a saturated air parcel during condensation imply a vertical displacement (O’Gorman 2011); if the distance to saturation is much larger in the horizontal than in the vertical direction (as is typically the case in an Earth-like climate), latent heat release becomes a function of local upward velocity and static stability,

$$Q_c = \omega^\uparrow \frac{\partial \rho \theta^\uparrow}{\partial p} \mathcal{H}(-\omega).$$ \hspace{2cm} (6)

Here, $\omega^\uparrow$ notes an upward velocity in pressure coordinates, $\frac{\partial \rho \theta^\uparrow}{\partial p}$ the static stability in updrafts, and $\mathcal{H}(-\omega)$ is the Heaviside function, which selectively removes downdrafts.

While O’Gorman (2011) considered the static stability in updrafts ($\frac{\partial \rho \theta^\uparrow}{\partial p}$) to depend on local thermodynamic conditions, in our study we take it to be in equilibrium with local near-surface conditions. That is, updrafts and condensation in the free troposphere are tied to the thermodynamic state of the boundary layer, with deep convection given by the convection scheme in our GCMs acting as communicator:

$$\frac{\partial \rho \theta^\uparrow}{\partial p} = \frac{\partial \rho \theta}{\partial p} |_{\theta_o^*}$$ \hspace{2cm} (7)

where $-\frac{\partial \rho \theta}{\partial p} |_{\theta_o^*}$ is the static stability set by air parcels lifted pseudo-adiabatically from the surface and conserving their near-surface moist entropy $\theta_o^*$ in the process. In the extratropics, deep baroclinic eddies set zones of deep ascent around cyclones or along frontal zones separating warm
subtropical air and cold polar air masses: there regions of ascent are co-located with vigorous
convective activity (Ralph et al. 2004).

While the net upward velocity ($\omega^\uparrow$) cannot be constrained without a detailed description of
the local energy budget, O’Gorman (2011) provides a geometric argument that relates the eddy
upward velocity approximately to the total eddy velocity:

$$\omega^\uparrow = \lambda \omega' + \varepsilon.$$  \hspace{1cm} (8)

Here $\lambda$ is a rescaling factor and $\varepsilon$ a residual. Relation (8) is satisfied identically when the eddy
velocity structure is either binary up and down, in which case $\lambda$ defines the surface area covered
by downdrafts. But relation (8) also is an adequate approximation for other (more realistic) eddy
velocity structures [see O’Gorman (2011) for details]. Dry eddies are expected to show equiparti-
tion between updrafts and downdrafts (i.e., $\lambda = 0.5$). In Earth’s atmosphere, NCEP-2 reanalyses
show that $\lambda$ converges to 1 in the deep tropics, where updrafts are confined to a small part of the
domain, while in the extratropics its value is near 0.55, consistent with near equipartition there
(O’Gorman 2011). This value was found to be nearly invariant in the extratropics over a wide
range of climates, which suggests that relation (8) is a useful representation of the eddy updraft
velocity (O’Gorman 2011).

Using near-convective neutrality to compute the static stability in updrafts (7) and the linearized
expression for eddy upward velocity (8), the eddy entropy budget may be expressed as

$$\partial_t \theta' = -\omega' \overline{\partial_p \theta^{\text{eff}}},$$  \hspace{1cm} (9)

where the effective static stability experienced by baroclinic eddies averaged over many lifecycles
is

$$\overline{\partial_p \theta^{\text{eff}}} = \partial_p \overline{\theta} - \lambda \partial_p \theta|_{\theta^*}.$$  \hspace{1cm} (10)
This effective static stability now incorporates the latent heat release (6) and thus makes the associated diabatic heating, which is external in dry dynamics, internal to the moist dynamics. An effective static stability measure similar to the dry static stability measure (4) can then be defined as

$$\Delta_{v}^{\text{eff}} \theta = -2 \overline{\partial_{p} \theta_{s}^{\text{eff}} (\bar{p}_{o} - \bar{p}_{l})}. \quad (11)$$

In a moist atmosphere, the effective static stability measure (11) is always smaller than the dry static stability measure (4), consistent with the added buoyancy provided by condensation and latent heat release. In a dry atmosphere, the static stability \( -\partial_{p} \theta \) equals its effective form \( -\partial_{p} \theta_{\text{eff}} \) because \( -\partial_{p} \theta \big|_{\theta^{*}_{o}} = 0 \). Whenever the convective lapse rate is more stable than its dry adiabatic limit, a reduced ’effective’ static stability is felt by eddies regardless of any other moisture effect. This has important implications for how eddies equilibrate in our dry GCM, as shown in section 4, as the convective lapse rates is prescribed to be moist-like to achieve greater resemblance with Earth’s climate.

With the effective static stability measure (11), an effective supercriticality \( S_{c}^{\text{eff}} \) can be defined, which is identical to relation (2) except for the replacement of the dry static stability by the effective moist static stability. The effective supercriticality is systematically greater than its dry form. O’Gorman (2011) found the effective supercriticality to be invariant at the terminus of the Hadley circulation, assuming a value \( S_{c}^{\text{eff}} \approx 0.6, \) in moist GCM simulations under global warming; this value is significantly larger than the dry supercriticality \( S_{c} \approx 0.4 \) found in the dry GCM simulations of Korty and Schneider (2008) or in the moist GCM simulations of Schneider and O’Gorman (2008) in the limit of vanishing moisture content.
3. Idealized GCMs

We examine whether the effective supercriticality and the constraint $S^\text{eff}_c \simeq 1$ can account for the extratropical thermal stratification and the extent of the Hadley circulation in a wide range of simulations with idealized GCMs. We use a dry GCM to investigate the effect of convective mixing on the supercriticality independent of other moisture effects (e.g., large-scale condensation). And we use a moist GCM to investigate more broadly how moist convection and moisture effects in baroclinic eddies affect the extratropical thermal structure and Hadley circulation extent.

a. Idealized dry GCM

The idealized dry GCM is described in Schneider and Walker (2006). It simulates an atmosphere bounded by a spatially uniform and thermally insulating spherical surface. A spectral dynamical core solves for the large-scale motions in the atmosphere with a resolution of T85 in all experiments. The vertical coordinate is discretized with 30 sigma levels. Momentum and dry entropy are diffused in a boundary layer of fixed height (2.5 km), with turbulent Prandtl number 1 (Smagorinsky et al. 1965). Frictional dissipation at the surface is parameterized by a bulk aerodynamic formula. Radiative heating is represented by Newtonian relaxation of potential temperatures toward a zonally symmetric radiative equilibrium profile, which is quadratic in the cosine of latitude and statically unstable to dry convection. The timescale for this Newtonian relaxation is zonally symmetric and varies with latitude and height from 7 days at the equator near the surface to 50 days in the interior atmosphere (Schneider 2004). Convection is parameterized by a quasi-equilibrium convection scheme. The convective lapse rate is varied by rescaling the dry adiabatic lapse rate by a factor $\gamma \leq 1$. Using a reduced convective lapse rate mimics changes in the stability of a convective atmosphere due to an increase in the near-surface moisture content: decreasing the rescaling factor $\gamma$ from its dry adiabatic value of 1 is similar to increasing the moisture content.
just above surface, which usually happens in a moist atmosphere as a response to warmer surface temperatures.

A wide range of climates is simulated by varying the pole-to-equator temperature contrast from 15 to 300 K and the convective lapse rate rescaling factor from 0.4 to 1.0. All 96 simulations were run for 2000 days. Statistics were accumulated after a statistically steady state was reached: they are averages over the last 600 days, sampled 4 times daily.

b. Idealized moist GCM

The idealized moist GCM is described in O’Gorman and Schneider (2008b). It simulates an atmosphere with an interactive hydrologic cycle, over a uniform ocean surface with a thermal inertia equivalent to one meter of water. The shortwave albedo of the planet is set to a uniform value, 0.38, which corresponds to the global-mean Bond albedo of Earth. Other constants (insolation, rotation rate, gravity, etc.) are kept the same as on Earth. A spectral dynamical core solves for large-scale motions in the atmosphere with a resolution of T42. As in the dry GCM, the vertical coordinate is discretized with 30 sigma levels. Water vapor is advected by the flow, and it condenses whenever saturation occurs on the grid scale. Momentum, moisture, and dry entropy fluxes at the surface are parameterized by bulk aerodynamic formulas; a k-profile scheme similar to Troen and Mahrt (1986) represents vertical transport by subgrid-scale motions. Moist convection is parameterized by a quasi-equilibrium convection scheme (O’Gorman and Schneider 2008b), similar to that in Frierson (2007). Radiative heating rates are computed for a gray atmosphere with no clouds, in which a prescribed longwave optical thickness roughly accounts for longwave absorption by water vapor and well-mixed greenhouse gases. The GCM is forced by an approximation of annual-mean insolation at the top of the atmosphere. This insolation is steady, i.e., there is no diurnal or seasonal cycle. The reference longwave optical thickness profile is chosen to lead to an extratropical
To obtain climate scenarios with a tropical climate and Hadley circulation resembling Earth’s, it is necessary to account for ocean heat transport in low-latitudes. We use a simple representation of ocean heat transport, which couples the heat transport to surface temperature gradients and surface wind stress (Klinger and Marotzke 2000; Levine and Schneider 2011).

We simulate a wide range of climates by varying the longwave optical depth and the insolation profile, in climate scenarios with and without the coupled ocean heat transport. The reference profile for the longwave optical depth is rescaled by a constant factor $\alpha$ to mimic changes in greenhouse gas concentrations; we use 7 different rescaling factors ranging from $\alpha = 0.2$ to 4.0. The insolation profile is varied by varying the pole-equator insolation contrast, while keeping the global mean fixed. We use between 14 and 19 insolation contrast values for every optical depth value, for a total of 109 climate simulations, each with and without coupled ocean heat transport. Some of these insolation contrast values are so large that insolation becomes negative at the poles; these simulations, despite not being realizable planetary configurations, extend our study to a climate range similarly wide as that covered by the dry GCM simulations. All simulations were run for 9 years. Statistics were accumulated over the last 4 years, from 4 times daily samples.

4. Results

This large ensemble of dry and moist scenarios (218 moist and 96 dry simulations) spans a very wide range of climates. In the moist GCM simulations, global-mean surface temperatures vary from 253 to 312 K, and pole-to-equator temperature contrasts from 11 to 144 K. We define a reference climate in this set of moist GCM simulations, characterized by a pole-to-equator surface temperature contrast (49 K) and a global-mean surface temperature (287 K) comparable to Earth’s climate in the annual-mean (e.g., O’Gorman and Schneider 2008a). In the dry GCM simulations,
global-mean surface temperatures vary from 243 to 385 K, and pole-to-equator temperature contrasts from 12 to 100 K.

As global-mean surface temperature increases, the tropopause shifts upward (Fig. 2), consistent with the radiative-convective response of an atmosphere to an increase in longwave opacity (Schneider 2007). Increasing pole-to-equator temperature contrasts also deepen the troposphere through an increase in static stability that, other factors equal, usually accompanies increased meridional temperature contrast. Deepening of the troposphere with temperature contrasts is especially large in cold simulations. This leads to a much weaker sensitivity of the tropopause height with global warming when temperature contrasts are large (Schneider 2007).

a. Supercriticality in midlatitudes

Schneider and Walker (2006) found $S_c \lesssim 1$ in baroclinic zones of dry GCM simulations, with $S_c < 1$ when convection played a substantial role in stabilizing the extratropical thermal stratification. Here we address whether the effective supercriticality, which takes the stabilizing effect of convection in baroclinic eddies into account, leads to a scaling $S_{c\text{eff}} \simeq 1$ over a wider range of climates in the dry GCM simulations and can also account for latent heat release in baroclinic eddies in the moist GCM simulations.

Figure 3 shows the static stability (4) vs. the meridional potential temperature contrast (3) in all dry and moist GCM simulations. Simulations with $S_c = 1$ lie on the 1-to-1 line.$^1$ Dry or moist simulations with dry or near-dry convective lapse rates (i.e., cold climates in moist simulations) lie near the line $S_c = 1$ (dark blue colors on Fig. 3). In them, the convective static stability is nearly zero, and the thermal stratification is stabilized by essentially dry baroclinic eddies. However,

$^1$Seven dry GCM simulations with low meridional potential temperature contrasts and low convective lapse rates were dominated by numerical noise. This prevented a robust estimate of baroclinic zones and of the Hadley circulation terminus. These simulations were excluded from Fig. 3 and all subsequent figures. In them, the tropopause height is controlled by convection.
these essentially dry simulations cover only a small subset of all simulations. For example, among the moist simulations, only those with global-mean surface temperatures below 270 K fall into this category. Most simulations, including the Earth-like reference simulation with the moist GCM, lie above the 1-to-1 line in Fig. 3. In these simulations, the static stability (4) is larger than the meridional potential temperature contrast (3), i.e., $S_c < 1$, and $S_c$ decreases as the convective lapse rate decreases (i.e., as colors in Fig. 3 shift from blue to red). A supercriticality $S_c \leq 1$ implies that convection does play a role in stabilizing the extratropical thermal stratification, although it may not act alone but in conjunction with baroclinic eddies, because the static stability is typically greater than that implied by convection alone (cf. Schneider and Walker 2006). For example, moist simulations with large meridional temperature contrasts are characterized by static stability values around 160 K (ordinate of Fig. 3), which exceeds the convective static stability, which is less than 120 K (colorbar in Fig. 3). In these simulations, the supercriticality does not provide information on the extratropical thermal stratification beyond the inequality constraint $S_c \lesssim 1$, and it does not discriminate effectively between purely convective and baroclinic-convective static stability regimes.

Using the effective static stability (11) provides additional information. Figure 4 shows the effective static stability (11) vs. the meridional potential temperature contrast (3). Now a much larger fraction of the simulations, both moist and dry, lie on the 1-to-1 line that signifies $S_{c,\text{eff}}^\text{eff} = 1$. Only in moist simulations with weak meridional temperature contrasts does convection control the extratropical thermal stratification ($S_{c,\text{eff}}^\text{eff} < 1$). In the moist simulations, the transition from baroclinic to convective regimes is temperature dependent, requiring greater baroclinicity as the

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2 Figure 4 shows 4 moist GCM simulations below the 1-to-1 line, i.e., with $S_{c,\text{eff}}^\text{eff} > 1$. These simulations correspond to very cold climates, with near-neutral thermal lapse rates. These simulations appear to be consistent with the findings of Jansen and Ferrari (2013), who also found $S_c > 1$ in simulations with near-neutral lapse rates.
climate warms: while the coldest moist simulations are controlled by baroclinic eddies regardless of their baroclinicities, in the warmest simulations, baroclinic eddies only dominate the extratropical thermal stratification when the meridional potential temperature contrast becomes larger than around 50K, which is when the meridional potential temperature contrast equals the effective static stability in radiative-convective equilibrium. Thus, the effective supercriticality can provide information on the extratropical thermal stratification even when convection and/or large-scale latent heat release in baroclinic eddies are important for the extratropical thermal stratification, as suggested by O’Gorman (2011) for moist simulations similar to those here. It can discriminate more effectively between purely convective and baroclinic-convective static stability regimes. Comparison of Figs. 4 and 3 shows that convection influences how baroclinic eddies affect the thermal stratification, even when turbulence is strong or when large-scale latent heat release has no effect of the dynamics.

b. Extent of Hadley circulation

Despite a large body of theoretical and observational studies, there is no unique definition for the terminus of the Hadley circulation (cf. Levine and Schneider 2011). Here, we define the terminus as the subtropical latitude where the mean meridional mass flux, integrated from the 750 hPa pressure level to the top of the atmosphere, attains 10% of its tropical extremum. In most climates, the 750 hPa pressure level is near the upper bound of the lower branch of the Hadley circulation. This definition is similar to others found in the literature (e.g., Lu et al. 2007). Figure 5 shows that the latitude of the terminus in both moist and dry simulations varies widely, from 8° to 36°. Dry GCM simulations suggest the existence of a maximum width for the Hadley circulation on an Earth-like planet: in simulations with large convective static stabilities and large meridional potential temperature contrasts, the terminus appears to asymptote toward 36°. One might think
this maximum latitude is given by the Held-Hou extent of angular-momentum conserving Hadley circulations (1). However, unlike the angular-momentum conserving extent, the terminus latitude in the dry simulations depends on the static stability, and it does not appear to continue to increase as the meridional potential temperature contrast increases.

In the moist simulations, the Hadley circulation widens as the climate warms, in agreement with comprehensive global warming simulations (e.g., Lu et al. 2007). The Hadley circulation terminus moves from 16° to 27° latitude when the longwave opacity in the simulations increases while the insolation is kept fixed (dashed lines in Fig. 5). The terminus also shifts poleward as the insolation contrast and with it meridional temperature contrasts increase: it moves from 16° to 26° when the insolation contrast is increased from zero to its maximum value consistent with nonnegative insolation at the poles, while the longwave opacity is kept fixed at its reference profile.

In the reference simulation without ocean heat transport (indicated by a pentagram in Fig. 5b), the Hadley circulation extends to 24° latitude. This is an about 6° smaller extent than on Earth’s in the annual mean. Once ocean heat transport is included, the Hadley circulation in the reference simulation is wider and comparable with Earth’s in the annual mean (compare Figs. 5b and c), consistent with the results of Levine and Schneider (2011).

In the dry simulations, Fig. 5 shows that the terminus shifts poleward as the pole-to-equator contrast in radiative equilibrium increases or the convective lapse rate decreases. These variations are qualitatively consistent with a widening of the circulation that accompanies a deepening of the troposphere or an increase in extratropical baroclinicity, as seen in the moist GCM simulations and in observations (Adam et al. 2014).
c. Supercriticality at the terminus of the Hadley circulation

In the simulations in which the thermal stratification in the extratropics is baroclinically controlled and \( S_{\text{eff}}^c \approx 1 \), the terminus of the Hadley circulation sits at the latitude at which \( S_{\text{eff}}^c \) first exceeds a critical \( O(1) \) value. That is, to the extent the effective supercriticality is a good measure of the depth of baroclinic eddies, the Hadley circulation extends to the latitude where baroclinic eddies become deep enough to reach the upper troposphere.

This can be seen in Fig. 6, which shows the effective static stability (4) vs. meridional entropy contrast (3) evaluated at the latitude of the terminus. Comparing these local quantities with the bulk quantities estimated over baroclinic zones (Fig. 4), we find very similar results. The only noticeable difference is the slope of the line around which simulations cluster: When baroclinic eddies control the extratropical thermal stratification (i.e., \( S_{\text{eff}}^c \approx 1.0 \) over the baroclinic zones), a lower but largely invariant value of supercriticality is found at the terminus (\( S_{\text{eff}}^c \approx 0.7 \)). This lower value of the supercriticality is conceptually consistent with the terminus being a transition between regions where baroclinic eddies dominate the dynamics (i.e., extratropical storm tracks) and regions where convection exerts a direct control on the tropospheric thermal stratification (i.e., the tropics). Similar results were obtained when using the dry static stability in lieu of the effective static stability, apart from an offset in the value of supercriticality at the terminus. That is, accounting for convection and large-scale condensation is not as important near the terminus than near the center of the baroclinic zones, apparently because convection is more prevalent in the storm track centers than in the subtropics.

5. Conclusions

Using a dry and a moist idealized GCM, we have simulated a wide range of climates. In the simulations, the terminus of the Hadley circulation varied between about 10° and 35° latitude.
A supercriticality criterion was modified to account for the tight coupling between baroclinic eddies and convection. This criterion successfully discriminated between simulations in which the extratropical thermal stratification was controlled by baroclinic eddies, and those in which it is controlled locally by convection. The latitude of the Hadley circulation terminus was found to be well constrained by a constant $O(1)$ value of the supercriticality whenever baroclinic eddies were strong; in this regime, the Hadley circulation terminus acts as the equatorward boundary for baroclinic wave activity that reaches the upper troposphere, as suggested in the dry context by Korty and Schneider (2008).

We found that the extratropical thermal stratification in idealized moist GCM simulations is controlled by baroclinic eddies when the meridional potential temperature contrast becomes larger than the effective static stability in local convective equilibrium. The Earth-like reference climate was found to be near the transition between baroclinic and convective regimes. More generally, we found convection to influence the static stability and tropopause height, even when baroclinic eddies were strong; this effect of convection was found to be greatly diminished at the Hadley circulation terminus compared to the center of baroclinic zones.

The finding that the effective supercriticality typically assumes a constant $O(1)$ value near the Hadley circulation termini offers an explanation for why the Hadley cells contract under El Niño but expand under La Niña and global warming (Seager et al. 2003; Lu et al. 2008; Tandon et al. 2013). Under El Niño, the equatorial atmosphere warms, leading to increased pole-equator temperature contrasts. For $S_c$ to assume a constant value of the Hadley circulation termini, the Hadley circulation needs to contract, provided changes in static stability are not large enough to compensate. Conversely, reduced pole-equator temperature contrasts under La Niña demand an expansion of the Hadley circulation. Similarly, under global warming, the static stability and tropopause height near the Hadley circulation termini generally increase, both ultimately because surface tem-
perature increases: this leads to increased static stability because of increased latent heat release (e.g., Frierson et al. 2006; Schneider and O’Gorman 2008) and an elevated tropopause because of higher temperatures and increased static stability (e.g., Schneider 2007). The supercriticality criterion for the extent of the Hadley circulation affords a unified and simple interpretation of these disparate phenomena.

Acknowledgments. We thank Paul O’Gorman for helpful clarifications on the effective static stability, Timothy M. Merlis for discussions on both linear baroclinic wave theories and ocean-atmosphere interactions, and Tobias Bischoff for his many helpful comments of this work and its relation to ENSO. We are grateful for support by the National Science Foundation (Grants AGS-1019211 and AGS-1049201) and a Yale Climate and Energy Institute Fellowship. The simulations were performed on the Division of Geological and Planetary Sciences’ Dell cluster at the California Institute of Technology. The program code for the simulations described in this paper is available at www.clidyn.ethz.ch/gcms/.

APPENDIX A

Determination of Baroclinic Zones

The supercriticality may be evaluated either locally (Eqn. 2) or as a bulk average. Schneider and Walker (2006) argued that the supercriticality should be estimated over a baroclinic zone that is at least as wide as the largest baroclinic eddies. Following Schneider and Walker (2006), we define baroclinic zones as regions where the near-surface eddy meridional potential temperature (heat) flux is greater than 50% of its maximum value. On Earth-like planets (in size and rotation rate), this region covers most of the extratropics, consistent with the deformation radius being of the scale of the planet. Schneider and Walker (2006) defined the center of the storm tracks as the
latitude where the eddy heat flux is maximum. In some climates, the eddy heat flux does not have a 
sharply defined maximum, which leads to sensitivity to the averaging convention when estimating 
baroclinic zones. We address this issue by defining the center of the storm track $\varphi_M$ from the eddy 
heat flux cumulative distribution, i.e.,

$$\varphi_M = \frac{\int_{\varphi^-}^{\varphi^+} (v' \theta') \, \varphi \, dp}{\int_{\varphi^-}^{\varphi^+} (v' \theta') \, dp}, \quad (A1)$$

Here, $\varphi^-$ and $\varphi^+$ define respectively the equatorward and poleward boundary of the baroclinic 
zones. A bulk estimate of supercriticality can then be defined as

$$\langle S_c \rangle = \frac{\langle \Delta_h \theta \rangle}{\langle \Delta_v \theta \rangle}, \quad (A2)$$

where $\langle \Delta_h \theta \rangle$ is defined as the bulk meridional potential temperature contrast,

$$\langle \Delta_h \theta \rangle = \frac{\int (\varphi_M) \, \beta(\varphi_M) \left[ \partial_y \bar{\theta}_s \right]}{\int (\varphi_M) \beta(\varphi_M)}, \quad (A3)$$

and $\langle \Delta_v \theta \rangle$ as the bulk static stability:

$$\langle \Delta_v \theta \rangle = -2 \left[ \partial_p \bar{\theta}_s \right] \left[ \bar{p}_o - \bar{p}_t \right]. \quad (A4)$$

Here, $\langle (\cdot) \rangle$ indicates a bulk estimate, and $\left[ (\cdot) \right]$ defines a meridional average over the baroclinic 
zone. Similarly, a bulk convective static stability can be defined as

$$\langle \Delta_v \theta \rangle = -2 \left[ \partial_p \bar{\theta}_s \right] \left[ \bar{p}_o - \bar{p}_t \right]. \quad (A5)$$

The bulk supercriticality characterizes whether baroclinic eddies control the thermal stratification: 
hence, $\langle S_c \rangle \sim 1$ if baroclinic entropy fluxes extend to the tropopause. If $\langle S_c \rangle \leq 1$, the tropopause 
height is controlled by convection; in this scenario, the bulk static stability contrast (A4) equals its 
convective value (A5).
Determining of Tropopause Height

A reliable determination of the tropopause height is essential when estimating the supercriticality in our simulations. The World Meteorological Organization defines the tropopause as the lowest level of the atmosphere where the lapse rate is equal or lower than a threshold lapse rate of 2 K km\(^{-1}\). We applied this definition to the bulk lapse rate (averaged over the baroclinic zones) for all climates. Tropopause heights estimated from this definition were then used to define the supercriticality (2). The same definition was used to define the local tropopause height in Fig. 2. We found that simulations with near-neutral convective lapse rate, which corresponds to cold climates in the moist GCM, have a poorly defined tropopause height when applying the WMO definition. To circumvent this issue, we use a different definition of the tropopause height based on the meridional circulation structure.

In the coldest set of simulations, the tropopause height was redefined as the level where the mean meridional mass flux in the upper troposphere reaches its global maximum; this global maximum is achieved in the deep tropics. This level usually corresponds to both levels of maximum eddy momentum flux divergence in the tropics and convergence in the extratropics, consistent with quasi-horizontal propagation of eddies from the storm tracks to the subtropics. The height obtained from this definition is nearly identical to that provided by the lapse rate definition except in the climates with near-neutral thermal stratification. In the latter simulations, the lapse rate definition fails to estimate the tropopause height, while our dynamical definition gives a more robust estimate.
References

Adam, O., T. Schneider, and N. Harnik, 2014: Role of changes in mean temperatures vs. temperature gradients in the recent widening of the Hadley circulation. *submitted to J. Climate*.


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Fig. 1. Cartoon of the effect of macroturbulence on the extent of the Hadley circulation. When the thermal contrast becomes large enough, baroclinic waves set the depth of the troposphere (tropopause shown by the red line): a change in the sign of the potential vorticity gradient between the surface layer (as shown by temperature decreasing poleward at the surface) and the interior of the troposphere (as shown by planetary vorticity increasing poleward) triggers baroclinic instability near the surface: eddies then propagate upward and equatorward into the subtropical upper troposphere, where they dissipate (blue wiggly arrows). The terminus of the Hadley circulation corresponds to a boundary between regions of eddy dissipation (subtropics) and eddy production (middle latitudes). In the upper troposphere, this is characterized by a transition between a region with eddy momentum flux divergence (orange patch) and poleward mean meridional winds (Hadley cell, solid black contours) to a region of eddy momentum flux convergence (blue patch) and equatorward mean meridional winds (Ferrel cell, dashed black contours).

Fig. 2. Mass flux streamfunction (clockwise: solid black lines; counter-clockwise: dashed black lines), mean potential temperature (cyan lines), mean zonal wind (magenta lines) and mean tropopause height as defined in the appendix (red solid line) for (a) a cold climate with low pole-to-equator temperature contrast (PETC), (b) a temperate climate with low PETC, (c) a warm climate with low PETC, (d) a cold climate with large PETC, (e) a temperate climate with large PETC, and (f) a warm climate with large PETC. The poleward and equatorward boundaries of the baroclinic zone as defined in the appendix are shown by the vertical green dashed lines. The global-mean surface temperature and the pole-to-equator temperature contrast of these simulations are shown in the bottom-right corner of the each panel. Contour intervals are 10 m s⁻¹ for zonal wind, 20 × 10⁶ kg s⁻¹ for streamfunction, and 10 K for potential temperature.

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Fig. 4. Meridional potential temperature contrast (3) and effective static stability (11) for the simulations shown in Fig. 3. Colors indicate the effective static stability in radiative-convective equilibrium. Other plotting conventions as in Fig. 3.

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